IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

In re Application of: Thomas Y-T. Tam et al.
Group Art Unit: 1732
Serial No: 10/699,416
Examiner: Patrick Butler
Filed: October 31, 2003
File No. H0004478 (4820)
For: PROCESS FOR DRAWING GEL-SPUN POLYETHYLENE YARNS

Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450

Sir:

DECLARATION UNDER 37 CFR § 1.132

I, Sheldon Kavesh, declare as follows:

I am presently an independent consultant. Honeywell International Inc. is a client of mine. Prior to 1999, I was a Senior Research Scientist in the Fiber Science Department of AlliedSignal Inc. (now Honeywell International Inc.) Morristown, New Jersey. I was the Chairman of the 1997 Gordon Research Conference on Fiber Science. I am a co-inventor of SPECTRA® high strength polyethylene fibers. My work on SPECTRA® was recognized by the awarding of a Gold Medal by the National Association for Science, Technology and Society, naming as an "Inventor of the Year" by the New Jersey Inventors Congress and Hall of Fame, and an IR-100 Award by IR Magazine (currently R&D Magazine). I am a named inventor of 57 United States Patents and an author of 14 publications in refereed journals related to fibers and materials. I was awarded the degree of Ph.D. in Chemical Engineering from the University of Delaware in 1968.

I have reviewed United States patent application Serial Number 10/699,416, filed October 31, 2003, which has been published as United States Patent Publication 20050093200 (the "Application"). I have also reviewed the Office Action dated November 16, 2006 in the Application (the "Office Action") and the references that were applied against the claims of the Application. I have been asked to respond to the following questions in relation to United States Patent 4,551,296 of which I am the principal inventor (and which has been cited against the claims of the Application):

1. What are the differences between drawing a multi-filament yarn in a tube under a nitrogen blanket and drawing in a forced convention air oven under turbulent conditions?
2. How does throughput capacity vary with the number of filaments in a yarn drawing process?
3. Is this different than for a spinning process?
4. In Example 523 of United States Patent 4,551,296, could the first stage draw ratio have been increased?
5. Are the mass throughputs achieved in United States Patent Publication 20050093200 what would be expected by one of skill in the art?

In response, I state as follows:

1. Polyethylene is a hydrocarbon polymer susceptible to oxidation and chain scission. The polyethylene fibers described in US 4,551,296 were drawn at elevated temperature under nitrogen blanketing to prevent oxidation, chain scission and expected reduction in tensile properties. That polyethylene fibers could be drawn to higher ratios and therefore drawn to higher strengths by drawing in air was not known or anticipated at the time of US 4,551,296. I know of no publication or patent that has since disclosed the advantage of drawing polyethylene in air as compared to drawing in nitrogen or other inert atmosphere.

   When drawing a yarn in a heated tube with essentially no forced gas flow as in my '296 patent, a laminar flow regime is established. The heat transmission rate to the exterior fibers in the yarn is about an order of magnitude lower than under turbulent gas flow conditions (see Appendix). Further, when tube drawing in a laminar flow regime, heat must

be conducted through the exterior fibers to reach the interior fibers. Temperature differences will exist from exterior to interior fibers. The consequence is non-uniform drawing stress across the yarn bundle, with resulting limitation on extent of draw. On the other hand, when a yarn is drawn in a turbulent gas stream, agitation of the yarn bundle exposes the interior fibers to the heated gases, minimizing temperature differences and permitting more uniform and higher draw. In my opinion, drawing of a polyethylene yarn in a forced convection oven under turbulent flow of air is not obvious from the teachings of any of the references cited against the claims in the Office Action.

2. For elevated temperature drawing, the time required to bring the interior of a yarn bundle up to the temperature of the environment increases as the square of the diameter of the yarn bundle. To achieve the same draw ratio with an increased number of filaments therefore generally necessitates slower feed speeds, lower draw ratios or both. However, that is not the end of the matter. As there is always some variation in filament-to-filament tenacity, the tenacity of a yarn generally decreases with increased numbers of filaments. If the criterion is constant yarn tenacity, an increased number of filaments may not increase the mass throughput of yarn in a drawing process. The relationship of yarn mass throughput to numbers of filaments will depend on many specific factors, including the fiber stress-strain characteristics, the filament-to-filament uniformity, as well as the drawing conditions. In my opinion, mass throughput in a drawing process near a limit of operability will not increase in simple proportion to the number of filaments. Consequently, it is incorrect to assume that the mass throughput of 16, 120 and 240 filaments would be the values as stated on Pages 6 and 10 of the Office Action.

3. In solution spinning of ultra-high molecular weight polyethylene, many of the same considerations apply as discussed with relation to the mass throughput capacity of a drawing process. In United States Patent 4,551,296, I state that the number of spinning apertures is not critical. However, that is not to imply that the mass throughput at some necessary level of filament quality will be proportional to the number of filaments even at the spinning stage.

4. The first stage draw ratio in Example 523 of United States Patent 4,551,296 was the maximum that could be run without filament breakage.

5. I was very surprised by the high mass throughputs that were achieved in the Application. The high mass throughputs evidently reflect higher heat transfer rates to the yarns and more uniform yarn temperatures as a result of drawing in a turbulent forced convection regime in air than I was able to obtain by tube drawing in essentially quiescent nitrogen. High mass throughputs provide a significant and practical advantage to the manufacturer in higher productivity and lower costs. In my opinion, the effect demonstrated in the Application is unexpected.

I certify that all statements made in this declaration made of my own knowledge are true and all statements made on information and belief are believed to be true.

(Willfully false statements and the like are punishable by fine or imprisonment, or both [18 U.S.C. 1001] and may jeopardize the validity of the application or any patent issuing thereon.)

Sheldon Kavesh
Sheldon Kavesh, Ph.D.

JAN. 10, 2007
Date

APPENDIX

Reference: "Perry's Chemical Engineers Handbook, Sixth Ed.", McGraw Hill Book Co., New York, 1984

I. Heat Transfer Under Laminar Flow in a Circular Tube, Constant Wall Temperature

The Nusselt Number is defined as hD/k. (Perry, P. 10-5)

where: h is the coefficient of heat transfer. cal/sq. cm-°C-sec
D is the diameter of the yarn bundle, cm
k is the thermal conductivity of air, cal/sq.cm--°C-sec/cm

In a laminar flow regime,

Nusselt Number (N_{Nu}) = 3.66 (Perry, Table 10-4, P. 10-15)

II. Heat Transfer Under Turbulent Flow in a Circular Tube Oven, Constant Wall Temperature

In a turbulent flow regime,

$N_{Nu} = 0.023\ (N_{Re})^{0.8}\ (N_{Pr})^{1/3}(\mu_b/\mu_w)^{0.14}$ for $N_{Re} > 10{,}000$ (Perry, Eq. 10.50, P. 10-16)

where: N_{Nu} is defined as above.
N_{Re} is the Reynolds Number of the flow.
N_{Pr} is the Prandtl Number of the heat transfer medium (air)
μ_b, μ_w are the viscosity of the heat transfer medium at the bulk temperature and at the wall temperature.
Prandtl Number (N_{Pr}) for air @ 151°C (304 °F) =0.69
$(N_{Pr})^{1/3} = 0.88$
Viscosity Ratio (Bulk/Wall)(μ_b/μ_w) ≈ 1

Substituting in Perry Eq. 10-50, we have
$N_{Nu} = hD/k > 0.023 \times (10{,}000)^{0.8} \times 0.88$
$hD/k > 32.1$
The ratio of the turbulent/laminar Nusselt numbers is:
$(hD/k)_{turbulent}/\ (hD/k)_{laminar} > 32.1/3.66 > 8.77$
Therefore, for constant yarn bundle diameter and at the same temperature (k=constant).

$h_{turbulent}/h_{laminar} > 8.77$

i.e., the heat transfer coefficient is at least about an order of magnitude greater in a turbulent flow regime than in a laminar flow regime.

PERRY'S CHEMICAL ENGINEERS' HANDBOOK

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Library of Congress Cataloging in Publication Data

Main entry under title:

Perry's Chemical engineers' handook.

(McGraw-Hill chemical engineering series)
Rev. ed. of: Chemical engineers' handbook. 5th ed. 1973.
Includes bibliographical references and index.
1. Chemical engineering—Handbooks, manuals, etc.
I. Perry, Robert H., 1924–1978. II. Green, Don W.
III. Maloney, James O. IV. Chemical engineers' handbook.
V. Series.
TP151.P45 1984 660.2'8 84-837

1234567890 DOW/DOW 8987654

ISBN 0-07-049479-7

The editors for this book were *Harold B. Crawford and Beatrice E. Eckes*, the designer was Mark E. Safran, and the production supervisor was Teresa M. Leaden. It was set in Caledonia by University Graphics, Inc.

Printed and bound by R. R. Donnelley & Sons Company.

Nomenclature and Units (*continued*)

Symbol	Definition	SI units	U.S. customary units
N_B	Biot number, $h_T\,\Delta x/k$		
N_d	Proportionality coefficient	Dimensionless	Dimensionless
N_{Gr}	Grashof number, $L^3\rho^2 g\beta\,\Delta t/\mu^2$		
N_{Nu}	Nusselt number, hD/k or hL/k		
N_{Pe}	Peclet number, DGc/k		
N_{Pr}	Prandtl number, $c\mu/k$		
N_{Re}	Reynolds number, DG/μ		
N_{St}	Stanton number, $N_{Nu}/N_{Re}N_{Pr}$		
N_{ss}	Number of sealing strips		
p	Pressure	kPa	lbf/ft^2 abs
p_f	Perimeter of a fin	m	ft
p'	Center-to-center spacing of tubes in tube bundle (tube pitch); p_n for tube pitch normal to flow; p_p for tube pitch parallel to flow	m	ft
Δp	Pressure of the vapor in a bubble minus saturation pressure of a flat liquid surface	kPa	lbf/ft^2 abs
P	Absolute pressure; P_c for critical pressure	kPa	lbf/ft^2
P'	Spacing between adjacent baffles on shell side of a heat exchanger (baffle pitch)	m	ft
ΔP_{bk}, ΔP_{wk}	Pressure drop for ideal-tube-bank cross-flow and ideal window respectively; ΔP_s for shell side of baffled exchanger	kPa	lbf/ft^2
q	Rate of heat flow, equals Q/θ	W, J/s	Btu/h
q'	Rate of heat generation	J/(s·m^3)	Btu/(h·ft^3)
$(q/A)_{max}$	Maximum heat flux in nucleate boiling	J/(s·m^2)	Btu/(h·ft^2)
Q	Quantity of heat; rate of heat transfer	J/s	Btu/h
Q	Quantity of heat; Q_T for total quantity	J	Btu
r	Radius; cylindrical and spherical coordinate; distance from midplane to a point in a body; r_1 for inner wall of annulus; r_2 for outer wall of annulus; r_i for inside radius of tube; r_m for distance from midplane or center of a body to the exterior surface of the body	m	ft
r_j	Inside radius	Dimensionless	Dimensionless
R	Thermal resistance, equals x/kA, $1/UA$, $1/hA$; R_1, R_2, R_3, R_n for thermal resistance of sections 1, 2, 3, and n of a composite body; R_T for sum of individual resistances of several resistances in series or parallel; R_{di} and R_{do} for dirt or scale resistance on inner and outer surface respectively	(s·K)/J	(h·°F)/Btu
R_f	Ratio of total outside surface of finned tube to area of tube having same root diameter		
S	Cross-sectional area; S_m for minimum cross-sectional area between rows of tubes, flow normal to tubes; S_{tb} for tube-to-baffle leakage area for one baffle; S_{sb} for shell-to-baffle area for one baffle; S_w for area for flow through window; S_{wg} for gross window area; S_{wt} for window area occupied by tubes	m^2	ft^2
S_r	Slope of rotary shell		
s	Specific gravity of fluid referred to liquid water		
t	Bulk temperature; temperature at a given point in a body at time θ	K	°F
t_1, t_2, t_n	Temperature at points 1, 2, and n in a system through which heat is being transferred	K	°F
t'	Temperature of surroundings	K	°F
t'_1, t'_2	Inlet and outlet temperature respectively of hotter fluid	K	°F
t''_1, t''_2	Inlet and outlet temperature respectively of colder fluid	K	°F
t_b	Initial uniform bulk temperature of a body; bulk temperature of a flowing fluid	K	°F
t_H, t_L	High and low temperature respectively on tube side of a heat exchanger	K	°F
t_s	Surface temperature	K	°F
t_{sv}	Saturated-vapor temperature	K	°F
t_w	Wall temperature	K	°F
t_∞	Temperature of undisturbed flowing stream	K	°F
T_H, T_L	High and low temperature respectively on shell side of a heat exchanger	K	°F

TABLE 10-2 Values of $(h_c + h_r)$*

Btu/(h·ft²·°F from pipe to room)
For horizontal bare standard steel pipe of various sizes in a room at 80°F

Nominal pipe diameter, in	Temperature difference, °F														
	30	50	100	150	200	250	300	350	400	450	500	550	600	650	700
1	2.16	2.26	2.50	2.73	3.00	3.29	3.60	3.95	4.34	4.73	5.16	5.60	6.05	6.51	6.98
3	1.97	2.05	2.25	2.47	2.73	3.00	3.31	3.69	4.03	4.43	4.85	5.26	5.71	6.19	6.66
5	....	1.95	2.15	2.36	2.61	2.90	3.20	3.54	3.90						
10	1.80	1.87	2.07	2.29	2.54	2.82	3.12	3.47	3.84						

*Bailey and Lyell [*Engineering*, 147, 60 (1939)] give values for $(h_c + h_r)$ up to Δt_s of 1000°F. °C = (°F − 32)/1.8; 5.6783 Btu/(h·ft²·°F) = J/(m²·s·/K).

Enclosed Spaces The rate of heat transfer across an enclosed space is calculated from a special coeffient h' based upon the temperature difference between the two surfaces, where $h' = (q/A)/(t_{s1} - t_{s2})$. The value of $h'L/k$ may be predicted from Eq. (10-32) by using the values of a and m given in Table 10-3.

TABLE 10-3 Values of a and m for Eq. (10-32)

Configuration	$N_{Gr}N_{Pr}(\delta/L)^3$	a	m
Vertical spaces	2×10^4 to 2×10^5	$0.20\,(\delta/L)^{-5/36}$	$\frac{1}{4}$
	2×10^5 to 10^7	$0.071\,(\delta/L)^{1/9}$	$\frac{1}{3}$
Horizontal spaces	10^4 to 3×10^5	$0.21\,(\delta/L)^{-1/4}$	$\frac{1}{4}$
	3×10^5 to 10^7	0.075	$\frac{1}{3}$

δ = cell width, L = cell length.

For vertical enclosed cells 10 in high and up to 2-in gap width, Landis and Yanowitz (*Proc. Third Int. Heat Transfer Conf.*, Chicago, 1966, vol. II, p. 139) give

$$q\delta/k\,\Delta t = 0.123(\delta/L)^{0.04}(N_{Gr}N_{Pr})^{0.28} \tag{10-35}$$

for $2 \times 10^3 < N_{Gr}N_{Pr}(\delta/L)^3 < 10^7$, where q is the uniform heat flux and Δt is the temperature difference at $L/2$. Equation (10-35) is applicable for air, water, and silicone oils.

For horizontal annuli Grugal and Hauf (*Proc. Third Int. Heat Transfer Conf.*, Chicago, 1966, vol. II, p 182) report

$$\frac{h\delta}{k} = \left(0.2 + 0.145\,\frac{\delta}{D_1}\,N_{Gr}\right)^{0.25} \exp\left(-0.02\,\frac{\delta}{D_1}\right) \tag{10-36}$$

for $0.55 < \delta/D_1 < 2.65$, where N_{Gr} is based upon gap width δ and D_1 is the core diameter of the annulus.

FORCED CONVECTION

Forced-convection heat transfer is the most frequently employed mode of heat transfer in the process industries. Hot and cold fluids, separated by a solid boundary, are pumped through the heat-transfer equipment, the rate of heat transfer being a function of the physical properties of the fluids, the flow rates, and the geometry of the system. Flow is generally turbulent, and the flow duct varies in complexity from circular tubes to baffled and extended-surface heat exchangers. Theoretical analyses of forced-convection heat transfer have been limited to relatively simple geometries and laminar flow. Analyses of turbulent-flow heat transfer have been based upon some mechanistic model and have not generally yielded relationships which were suitable for design purposes. Usually for complicated geometries only empirical relationships are available, and frequently these are based upon limited data and special operating conditions. Heat-transfer coefficients are strongly influenced by the mechanics of flow occurring during forced-convection heat transfer. Intensity of turbulence, entrance conditions, and wall conditions are some of the factors which must be considered in detail as greater accuracy in prediction of coefficients is required.

Analogy between Momentum and Heat Transfer The interrelationship of momentum transfer and heat transfer is obvious from examining the equations of motion and energy. For constant fluid properties, the equations of motion must be solved before the energy equation is solved. If fluid properties are not constant, the equations are coupled, and their solutions must proceed simultaneously. Considerable effort has been directed toward deriving some simple relationship between momentum and heat transfer. The methodology has been to use easily observed velocity profiles to obtain a measure of the diffusivity of momentum in the flowing stream. The analogy between heat and momentum is invoked by assuming that diffusion of heat and diffusion of momentum occur by essentially the same mechanism so that a relatively simple relationship exists between the diffusion coefficients. Thus, the diffusivity of momentum is used to predict temperature profiles and thence by Eq. (10-25) to predict the heat-transfer coefficient.

The analogy has been reasonably successful for simple geometries and for fluids of very low Prandtl number (liquid metals). For high-Prandtl-number fluids the empirical analogy of Colburn [*Trans. Am. Inst. Chem. Eng.*, 29, 174 (1933)] has been very successful. A j factor for momentum transfer is defined as $j = f/2$, where f is the friction factor for the flow. The j factor for heat transfer is assumed to be equal to the j factor for momentum transfer

$$j = h/cG(c\mu/k)^{2/3} \tag{10-37}$$

More involved analyses for circular tubes reduce the equations of motion and energy to the form

$$\frac{\tau g_c}{\rho} = -\frac{(\nu + \epsilon_M)\,du}{dy} \tag{10-38a}$$

$$\frac{q/A}{c\rho} = -\frac{(\alpha + \epsilon_H)\,dt}{dy} \tag{10-38b}$$

where ϵ_H is the eddy diffusivity of heat and ϵ_M is the eddy diffusivity of momentum. The units of diffusivity are L^2/θ. The eddy viscosity is $E_M = \rho\epsilon_M$, and the eddy conductivity of heat is $E_H = \epsilon_H c\rho$. Values of ϵ_M are determined via Eq. (10-38a) from experimental velocity-distribution data. By assuming ϵ_H/ϵ_M = constant (usually unity), Eq. (10-38b) is solved to give the temperature distribution from which the heat-transfer coefficient may be determined. The major difficulties in solving Eq. (10-38b) are in accurately defining the thickness of the various flow layers (laminar sublayer and buffer layer) and in obtaining a suitable relationship for prediction of the eddy diffusivities. For assistance in predicting eddy diffusivities, see Reichardt (NACA Tech. Memo 1408, 1957) and Strunk and Chao [*Am. Inst. Chem. Eng. J.*, 10, 269 (1964)].

Internal and External Flow Two main types of flow are considered in this subsection: internal or conduit flow, in which the fluid completely fills a closed stationary duct, and external or immersed flow, in which the fluid flows past a stationary immersed solid. With internal flow, the heat-transfer coefficient is theoretically infinite at the location where heat transfer begins. The local heat-transfer coefficient rapidly decreases and becomes constant, so that after a certain length the average coefficient in the conduit is independent of the length. The local coefficient may follow an irregular pattern, however, if obstructions or turbulence promoters are present in the duct. For immersed flow, the local coefficient is again infinite at the point where heating begins, after which it decreases and may show various irregularities depending upon the configuration of the body. Usually in this instance the local coefficient never becomes constant as flow proceeds downstream over the body.

When heat transfer occurs during immersed flow, the rate is dependent upon the configuration of the body, the position of the body, the proximity of other bodies, and the flow rate and turbulence of the stream. The heat-transfer coefficient varies over the immersed body, since both the thermal and the momentum boundary layers vary in thickness. Relatively simple relationships are available for simple configurations immersed in an infinite flowing fluid. For complicated configurations and assemblages of bodies such as are found on the shell side of a heat exchanger, little is known about the local heat-transfer coefficient; empirical relationships giving average coefficients are all that are usually available. Research that has been conducted on local coefficients in complicated geometries has not been extensive enough to extrapolate into useful design relationships.

Laminar Flow Normally, laminar flow occurs in closed ducts when $N_{Re} < 2100$ (based on equivalent diameter $D_e = 4 \times$ free area ÷ perimeter). Laminar-flow heat transfer has been subjected to extensive theoretical study. The energy equation has been solved for a variety of boundary conditions and geometrical configurations. However, true laminar-flow heat transfer very rarely occurs. Natural-convection effects are almost always present, so that the assumption that molecular conduction alone occurs is not valid. Therefore, empirically derived equations are most reliable.

Data are most frequently correlated by the Nusselt number $(N_{Nu})_{lm}$ or $(N_{NU})_{am}$, the Graetz number $N_{Gz} = (N_{Re}N_{Pr}D/L)$, and the Grashof (natural-convection effects) number N_{Gr}. Some correlations consider only the variation of viscosity with temperature, while others also consider density variation. Theoretical analyses indicate that for very long tubes $(N_{Nu})_{lm}$ approaches a limiting value. Limiting Nusselt numbers for various closed ducts are shown in Table 10-4.

TABLE 10-4 Values of Limiting Nusselt Number in Laminar Flow in Closed Ducts

Configuration	Limiting Nusselt number $N_{Gr} < 4.0$	
	Constant wall temperature	Constant heat flux
Circular tube	3.66	4.36
Concentric annulus	....	Eq. (10-42)
Equilateral triangle	....	3.00
Rectangles		
Aspect ratio:		
1.0 (square)	2.89	3.63
0.713	....	3.78
0.500	3.39	4.11
0.333	....	4.77
0.25	....	5.35
0 (parallel planes)	7.60	8.24

Circular Tubes For horizontal tubes several relationships are applicable, depending upon the value of the Graetz number. For $N_{Gz} < 100$, Hausen's [*Z. Ver. Dtsch. Beth. Verfahrenstech.*, no. 4, 91 (1943)] equation is recommended:

$$(N_{Nu})_{lm} = 3.66 + \frac{0.085N_{Gz}}{1 + 0.047N_{Gz}^{2/3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \qquad (10\text{-}39)$$

For $N_{Gz} > 100$, the Sieder-Tate relationship [*Ind. Eng. Chem.*, 28, 1429 (1936)] is satisfactory for small diameters and Δt's:

$$(N_{Nu})_{am} = 1.86N_{Gz}^{1/3}(\mu_b/\mu_w)^{0.14} \qquad (10\text{-}40)$$

A more general expression covering all diameters and Δt's is obtained by including an additional factor $0.87(1 + 0.015N_{Gr}^{1/3})$ on the right side of Eq. (10-40). The diameter should be used in evaluating N_{Gr}. An equation published by Oliver [*Chem. Eng. Sci.*, 17, 335 (1962)] is also recommended.

For laminar flow in vertical tubes a series of charts developed by Pigford [*Chem. Eng. Prog. Symp. Ser.* 17, 51, 79 (1955)] may be used to predict values of h_{am}.

Annuli Approximate heat-transfer coefficients for laminar flow in annuli may be predicted by the equation of Chen, Hawkins, and Solberg [*Trans. Am. Soc. Mech. Eng.*, 68, 99 (1946)]:

$$(N_{Nu})_{am} = 1.02N_{Re}^{0.45}N_{Pr}^{0.5}\left(\frac{D_e}{L}\right)^{0.4}\left(\frac{D_2}{D_1}\right)^{0.8}\left(\frac{\mu_b}{\mu_1}\right)^{0.14}N_{Gr}^{0.05} \qquad (10\text{-}41)$$

Limiting Nusselt numbers for slug-flow annuli may be predicted (for constant heat flux) from Trefethen (*General Discussions on Heat Transfer*, London, ASME, New York, 1951, p. 436):

$$(N_{Nu})_{lm} = \frac{8(m-1)(m^2-1)^2}{4m^4 \ln m - 3m^4 + 4m^2 - 1} \qquad (10\text{-}42)$$

where $m = D_2/D_1$. The Nusselt and Reynolds numbers are based on the equivalent diameter, $D_2 - D_1$.

Limiting Nusselt numbers for laminar flow in annuli have been calculated by Dwyer [*Nucl. Sci. Eng.*, 17, 336 (1963)]. In addition, theoretical analyses of laminar-flow heat transfer in concentric and eccentric annuli have been published by Reynolds, Lundberg, and McCuen [*Int. J. Heat Mass Transfer*, 6, 483, 495 (1963)]. Lee [*Int. J. Heat Mass Transfer*, 11, 509 (1968)] presented an analysis of turbulent heat transfer in entrance regions of concentric annuli. Fully developed local Nusselt numbers were generally attained within a region of 30 equivalent diameters for $0.1 < N_{Pr} < 30$, $10^4 < N_{Re} < 2 \times 10^5$, $1.01 < D_2/D_1 < 5.0$.

Parallel Plates and Rectangular Ducts The limiting Nusselt number for parallel plates and flat rectangular ducts is given in Table 10-4. Norris and Streid [*Trans. Am. Soc. Mech. Eng.*, 62, 525 (1940)] report for constant wall temperature

$$(N_{Nu})_{lm} = 1.85N_{Gz}^{1/3} \qquad (10\text{-}43)$$

for $N_{Gz} > 70$. Both Nusselt number and Graetz numbers are based on equivalent diameter. For large temperature differences it is advisable to apply the correction factor $(\mu_b/\mu_w)^{0.14}$ to the right side of Eq. (10-43).

For rectangular ducts Kays and Clark (Stanford Univ., Dept. Mech. Eng. Tech. Rep. 14, Aug. 6, 1953) published relationships for heating and cooling of air in rectangular ducts of various aspect ratios. For most noncircular ducts Eqs. (10-39) and (10-40) may be used if the equivalent diameter (= 4 × free area/wetted perimeter) is used as the characteristic length. See also Kays and London, *Compact Heat Exchangers*, 2d ed., McGraw-Hill, New York, 1964.

Immersed Bodies When flow occurs over immersed bodies such that the boundary layer is completely laminar over the whole body, laminar flow is said to exist even though the flow in the mainstream is turbulent. The following relationships are applicable to single bodies immersed in an infinite fluid and *are not valid for assemblages of bodies.*

In general, the average heat-transfer coefficient on immersed bodies is predicted by

$$N_{Nu} = C_r(N_{Re})^m(N_{Pr})^{1/3} \qquad (10\text{-}44)$$

Values of C_r and m for various configurations are listed in Table 10-5. The characteristic length is used in both the Nusselt and the Reynolds numbers, and the properties are evaluated at the film temperature $= (t_w + t_\infty)/2$. The velocity in the Reynolds number is the undisturbed free-stream velocity.

Heat transfer from immersed bodies is discussed in detail by Eckert and Drake, Jakob, and Knudsen and Katz (see "Introduction: General References"), where equations for local coefficients and the effects of unheated starting length are presented. Equation (10-44) may also be expressed as

$$N_{St}N_{Pr}^{2/3} = C_rN_{Re}^{m-1} = f/2 \qquad (10\text{-}45)$$

where f is the *skin-friction drag coefficient* (not the form drag coefficient).

Falling Films When a liquid is distributed uniformly around the periphery at the top of a vertical tube (either inside or outside) and allowed to fall down the tube wall by the influence of gravity, the fluid does not fill the tube but rather flows as a thin layer. Similarly, when a liquid is applied uniformly to the outside and top of a horizontal tube, it flows in layer form around the periphery and falls

TABLE 10-5 Laminar-Flow Heat Transfer over Immersed Bodies [Eq. (10-44)]

Configuration	Characteristic length	N_{Re}	N_{Pr}	C_r	m
Flat plate parallel to flow	Plate length	10^3 to 3×10^5	>0.6	0.648	0.50
Circular cylinder axes perpendicular to flow	Cylinder diameter	1–4		0.989	0.330
		4–40		0.911	0.385
		40–4000	>0.6	0.683	0.466
		4×10^3–4×10^4		0.193	0.618
		4×10^4–2.5×10^5		0.0266	0.805
Non-circular cylinder, axis	Square, short diameter	5×10^3–10^5		0.104	0.675
Perpendicular to flow, characteristic	Square, long diameter	5×10^3–10^5		0.250	0.588
Length perpendicular to flow	Hexagon, short diameter	5×10^3–10^5	>0.6	0.155	0.638
	Hexagon, long diameter	5×10^3–2×10^4		0.162	0.638
		2×10^4–10^5		0.0391	0.782
Sphere*	Diameter	1–7×10^4	0.6–400	0.6	0.50

*Replace N_{Nu} by $N_{Nu} - 2.0$ in Eq. (10-44).

off the bottom. In both these cases the mechanism is called gravity flow of liquid layers or falling films.

For the turbulent flow of water in layer form down the walls of vertical tubes the dimensional equation of McAdams, Drew, and Bays [*Trans. Am. Soc. Mech. Eng.*, **62**, 627 (1940)] is recommended:

$$h_{lm} = b\Gamma^{1/3} \tag{10-46}$$

where $b = 9150$ (SI) or 120 (U.S. customary) and is based on values of $\Gamma = W_F/\pi D$ ranging from 0.25 to 6.2 kg/(m·s) [600 to 15,000 lb/(h·ft)] of wetted perimeter. This type of water flow is used in vertical vapor-in-shell ammonia condensers, acid coolers, cycle water coolers, and other process-fluid coolers.

The following dimensional equations may be used for any liquid flowing in layer form down vertical surfaces:

$$\text{For } \frac{4\Gamma}{\mu} > 2100 \quad h_{lm} = 0.01\left(\frac{k^3\rho^2 g}{\mu^2}\right)^{1/3}\left(\frac{c\mu}{k}\right)^{1/3}\left(\frac{4\Gamma}{\mu}\right)^{1/3} \tag{10-47a}$$

$$\text{For } \frac{4\Gamma}{\mu} < 2100 \quad h_{am} = 0.50\left(\frac{k^2\rho^{4/3}cg^{2/3}}{L\mu^{1/3}}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{1/4}\left(\frac{4\Gamma}{\mu}\right)^{1/9} \tag{10-47b}$$

where $\quad B = (3\mu\Gamma/\rho^2 g)^{1/3}$

Equation (10-47*b*) is based on the work of Bays and McAdams [*Ind. Eng. Chem.*, **29**, 1240 (1937)]. The significance of the term L is not clear. When $L = 0$, the coefficient is definitely not infinite. When L is large and the fluid temperature has not yet closely approached the wall temperature, it does not appear that the coefficient should necessarily decrease. Within the finite limits of 0.12 to 1.8 m (0.4 to 6 ft), this equation should give results of the proper order of magnitude.

For falling films applied to the outside of horizontal tubes, the Reynolds number rarely exceeds 2100. Equations may be used for falling films on the outside of the tubes by substituting $\pi D/2$ for L.

For water flowing over a horizontal tube, data for several sizes of pipe are roughly correlated by the dimensional equation of McAdams, Drew, and Bays [*Trans. Am. Soc. Mech. Eng.*, **62**, 627 (1940)].

$$h_{am} = b\,(\Gamma/D_0)^{1/3} \tag{10-48}$$

where $b = 3360$ (SI) or 65.6 (U.S. customary) and Γ ranges from 0.94 to 4 kg/m·s) [100 to 1000 lb/(h·ft)].

Falling films are also used for evaporation in which the film is both entirely or partially evaporated (juice concentration). This principle is also used in crystallization (freezing).

The advantage of high coefficient in falling-film exchangers is partially offset by the difficulties involved in distribution of the film, maintaining complete wettability of the tube, and pumping costs required to lift the liquid to the top of the exchanger.

Transition Region Turbulent-flow equations for predicting heat transfer coefficients are usually valid only at Reynolds numbers greater than 10,000. The transition region lies in the range $2000 < N_{Re} < 10{,}000$. No simple equation exists for accomplishing a smooth mathematical transition from laminar flow to turbulent flow. Of the relationships proposed, Hausen's equation [*Z. Ver. Dtsch. Ing. Beih. Verfahrenstech.*, No. 4, 91 (1934)] fits both the laminar extreme and the fully turbulent extreme quite well.

$$(N_{Nu})_{am} = 0.116(N_{Re}^{2/3} - 125)N_{Pr}^{1/3}\left[1 + \left(\frac{D}{L}\right)^{2/3}\right]\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \tag{10-49}$$

between 2100 and 10,000. It is customary to represent the probable magnitude of coefficients in this region by hand-drawn curves (Fig. 10-8). Equation (10-40) is plotted as a series of curves (*j* factor versus

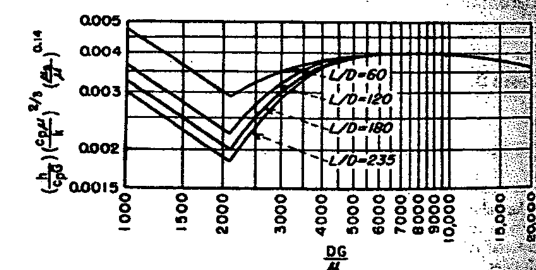

FIG. 10-8 Graphical representation of the Colburn *j* factor for the heating and cooling of fluids inside tubes. The curves for N_{Re} below 2100 are based on Eq. (10-40). L is the length of each pass in feet. The curves for N_{Re} between 2100 and 10,000 are represented by Eq. (10-49). The curve for N_{Re} above 10,000 is represented by Eq. (10-51).

Reynolds number with L/D as parameters) terminating at Reynolds number = 2100. Continuous curves for various values of L/D are then hand-drawn from these terminal points to coincide tangentially with the curve for forced-convection, fully turbulent flow [Eq. (10-51)].

Turbulent Flow

Circular Tubes Numerous relationships have been proposed for predicting turbulent flow in tubes. For high-Prandtl-number fluids, relationships derived from the equations of motion and energy through the momentum–heat-transfer analogy are more complicated and no more accurate than many of the empirical relationships that have been developed.

For $N_{Re} > 10{,}000$, $0.7 < N_{Pr} < 700$, $L/D > 60$ and properties based on bulk temperature, the Sieder-Tate equation is recommended:

$$N_{Nu} = 0.023N_{Re}^{0.8}N_{Pr}^{1/3}\,(\mu_b/\mu_w)^{0.14} \tag{10-50}$$

The Colburn form of Eq. (10-50) is

$$j_H = N_{St}N_{Pr}^{2/3}(\mu_w/\mu_b)^{0.14} = 0.023N_{Re}^{-0.2} \tag{10-51}$$

In Eq. (10-51) the viscosity-ratio factor may be neglected if properties are evaluated at the film temperature $(t_b + t_w)/2$.